Semi - Empirical Mass Formula

The mass of a neutral atom whose nucleus contains z protons and (A-z) neutrons is given by

$$ {}^{A}_{z}M = zM_p + (A-z)M_n + zM_e - B $$
$$ = zM_H + (A-z)M_n - B, $$

where B is the binding energy.

Von Weizsacker in 1935 obtained a formula, known as semi-empirical mass formula, for expressing the atomic mass of a nuclide, in terms of a series of binding energy terms. we shall now proceed term by term.

(a) Volume binding energy :→ Since the volume of the nucleus is proportional to A, hence this term may be regarded as a volume binding energy, and is given by

$$ B_v = a_v A $$

where a_v is a proportionality constant.

(b) Surface binding energy :→ The main binding energy must be corrected by a disruptive term, propotional to the surface area, known as surface energy. It is proportional to R^2 or $A^{2/3}$.

$$B_S = -a_S A^{2/3}$$

where a_S is the proportionality constant.

(c) Coulomb Energy :→ This energy arises due to coulomb repulsion b/w protons. The work in bringing this charge dq from infinity to r against the charge on the sphere of radius r is

$$\frac{3Z^2e^2}{4\pi\varepsilon_0 R_0^6 A^2} r^4 dr.$$

Integrating from 0 to $R(=R_0A^{1/3})$. the loss of binding energy B_C due to the disruptive Coulomb energy.

$$B_C = -\frac{3}{5}\frac{Z^2e^2}{4\pi\varepsilon_0 R_0 A^{1/3}} = -a_C\frac{Z^2}{A^{1/3}}$$

where a_C is a constant and the subscript c is for Coulomb.

(d) Asymmetry Energy :→ Heavy nuclie have more neutrons than protons. A simple theory of this asymmetry is shown in schematic diagram of nuclear energy levels. The transfer of nuclons decreases the binding energy. Hence decrease in binding energy due to asymmetry

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$$B_{as} = -a_a\frac{(N-Z)^2}{A} = -a_a\frac{(A-2Z)^2}{A},$$

where a_a is a constant and subscript a is for asymmetry.

(e) Pairing Energy :→ The nuclides with even numbers of protons and neutrons are most stable. Nuclei with odd numbers of both neutrons and protons are the least stable, while nuclei for which either proton or neutron number is odd are intermediate in stability. To take account of this pairing effect an additional term is used. We have $\delta = a_p A^{-3/4}$, where a_p is the pairing energy constant. Hence we have

$B_p = 0$ for A odd, Z even and N odd ~~or~~ Z odd and Z even,

$B_p = a_p/A^{3/4}$, for A even, Z even and N even.

$B_p = -a_p/A^{3/4}$, for A even, Z odd and N odd.

Collecting together the mass correction terms we can write Weizsacker's semi-empirical mass formula as

$$\boxed{{}^{A}_{Z}M = ZM_H + (A-Z)M_n - a_v A + a_s A^{2/3} + a_c Z^2 A^{-1/3} + a_a(A-2Z)^2 A^{-1} \mp a_p A^{-3/4}}$$

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